Quantum Perspectivism and Its Relations: RQM, QBism, Many-Worlds, and Topos Theory

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Abstract

We provide a comprehensive comparison of Quantum Perspectivism (QP)—the framework in which quantum mechanics emerges from the Yoneda Lemma as a universal structural constraint—with the major existing interpretive and mathematical programs in quantum foundations. We examine in detail the relationship to Rovelli's Relational Quantum Mechanics (RQM), showing that QP shares its commitment to relationality but grounds it in the rigorous mathematics of the Yoneda embedding rather than leaving the relational ontology informal and under-determined. We compare with QBism (Quantum Bayesianism), demonstrating that while both approaches reject observer-independent quantum states, QP replaces subjective Bayesian credences with objective categorical structure in the presheaf topos. The Everettian Many-Worlds Interpretation is analyzed as a framework that preserves unitarity at the cost of ontological proliferation, whereas QP achieves the same formal unitarity without additional worlds by treating definiteness as context-relative restriction of a presheaf. We provide an extensive treatment of the Isham–Butterfield–Döring topos-theoretic program, showing that QP supplies the physical motivation their approach lacked, and we examine the connections to the Abramsky–Coecke categorical quantum mechanics program. We further extend the comparison to consistent and decoherent histories (Griffiths, Omnès, Gell-Mann–Hartle), objective collapse theories (GRW, Penrose), the de Broglie–Bohm pilot wave theory, and the transactional interpretation, demonstrating in each case how the Yoneda Constraint clarifies, subsumes, or diverges from these approaches. A systematic comparison table is provided.

Keywords: Yoneda Lemma, quantum foundations, relational quantum mechanics, QBism, many-worlds interpretation, topos quantum theory, categorical quantum mechanics, presheaf, Kochen–Specker theorem, measurement problem

Contents

1 Introduction

Quantum mechanics, despite nearly a century of unbroken empirical success, remains the subject of deep foundational disagreement. The landscape of interpretations—from Copenhagen to Many-Worlds, from hidden variables to information-theoretic approaches—continues to expand, with each framework offering partial insight but none achieving universal acceptance. This situation is not merely a philosophical curiosity; the choice of foundational framework has real consequences for the development of quantum gravity, quantum information theory, and the conceptual training of the next generation of physicists.

In a companion paper [1], we introduced **Quantum Perspectivism** (QP), a framework in which the structures of quantum mechanics—Hilbert spaces, the Born rule, superposition, entanglement, complementarity, and the measurement problem—emerge as direct consequences of the **Yoneda Constraint**: the requirement, grounded in the Yoneda Lemma of category theory, that physical systems are completely characterized by their relational profiles. The Yoneda embedding

$$\mathsf{y} : \mathscr{C} \hookrightarrow [\mathscr{C}^{\mathrm{op}}, \mathbf{Set}] \tag{1}$$

maps each observational context into the topos of presheaves, and the full faithfulness of this embedding ensures that no physical information is lost in the perspectival description.

The purpose of the present paper is to situate Quantum Perspectivism within the existing landscape of quantum foundations. We aim to show that QP is not merely another interpretation added to an already crowded field, but rather a *unifying meta-framework* that clarifies why each existing approach captures part of the truth, identifies what each gets right, diagnoses what each gets wrong, and demonstrates how the Yoneda Constraint provides the missing structural backbone that resolves the tensions between them.

Our approach is systematic. For each framework, we (i) recall its essential claims and formal structure, (ii) identify the precise points of agreement and disagreement with QP, (iii) demonstrate how the Yoneda Constraint either subsumes, extends, or diverges from the framework, and (iv) assess the philosophical and technical advantages of the QP perspective. We organize the comparisons in order of increasing formal distance from QP, beginning with Relational Quantum Mechanics (which shares the most conceptual DNA) and ending with objective collapse theories (which are most distant in spirit).

The paper is structured as follows. Section 2 provides a brief recap of the core structures of Quantum Perspectivism. Sections 3 through 9 develop the detailed comparisons. Section 10 provides a systematic comparison table. Section 11 offers a synthesis and discusses the extent to which QP provides a genuine unification. Section 12 concludes.

2 Recap: The Core of Quantum Perspectivism

We briefly recall the essential elements of Quantum Perspectivism as developed in [1]; the reader is referred to that paper for complete proofs and detailed constructions.

2.1 The Yoneda Constraint

The **Yoneda Lemma** establishes that for any category $\mathscr{C}$, object $A \in \mathscr{C}$, and presheaf $F : \mathscr{C}^{\mathrm{op}} \to \mathbf{Set}$:

$$\mathrm{Nat}(\mathsf{y}(A), F) \cong F(A), \tag{2}$$

naturally in A and F. The Yoneda embedding $\mathsf{y} : \mathscr{C} \hookrightarrow \widehat{\mathscr{C}}$ is fully faithful, meaning that objects are completely determined by their relational profiles.

Axiom 1 (The Yoneda Constraint)**.** A physical system S is completely determined by its relational profile: the totality of morphisms from all possible probe systems into S. There are no physical properties of S beyond those accessible via such morphisms.

2.2 Physical Systems as Presheaves

Let $\mathscr{C}$ be the category of **observational contexts**—complete specifications of experimental setups, reference frames, and measurement configurations—with morphisms representing refinements or transformations between contexts. A physical system is a presheaf

$$S : \mathscr{C}^{\mathrm{op}} \to \mathbf{Set}, \tag{3}$$

assigning to each context C a set $S(C)$ of outcomes or appearances, and to each refinement $f : C' \to C$ a restriction map $S(f) : S(C) \to S(C')$.

2.3 The Derivation Chain

The key results established in [1] form a derivation chain:

(i) **Monoidal structure on** $\mathscr{C}$ forces the fibers $S(C)$ to carry vector space structure over a field k.

(ii) **Braided monoidal structure with fermionic sector** forces $k = \mathbb{C}$.

(iii) **Perspectival consistency** yields a Hermitian inner product, making each $S(C)$ a Hilbert space $\mathcal{H}_C$.

(iv) **Observables** are self-adjoint natural transformations $\alpha : S \Rightarrow S$.

(v) **The Born rule** follows from the Yoneda isomorphism combined with Gleason's theorem.

(vi) **Measurement** is presheaf restriction along a morphism—no collapse.

(vii) The **presheaf topos** $\widehat{\mathscr{C}}$ provides quantum logic via its subobject classifier.

2.4 Key Philosophical Commitments

QP is committed to:

- **Ontic structural realism**: the fundamental ontology is relational structure, not substance.
- **Perspectival objectivity**: the presheaf is an objective mathematical entity, but its content is inherently perspectival.

- **No collapse**: measurement is context-restriction, not a physical process.
- **No hidden variables**: the Yoneda embedding is fully faithful, leaving no room for properties beyond the relational profile.

3 Relational Quantum Mechanics

3.1 Rovelli's Program: Core Principles

Carlo Rovelli's Relational Quantum Mechanics (RQM), first articulated in 1996 [2], represents perhaps the most significant attempt to take the relational character of quantum mechanics seriously at the foundational level. RQM rests on two central theses:

(R1) **Completeness of quantum mechanics**: Quantum mechanics provides a complete description of the physical world; no additional variables or structures are needed.

(R2) **Relationality of quantum states**: The quantum state ψ of a system S is not an absolute property of S but is always defined *relative to* another physical system O (the "observer" or reference system). Different reference systems may assign different quantum states to the same system without contradiction.

Rovelli draws an analogy with special relativity, where the simultaneity of distant events is observer-relative. In RQM, the definiteness of quantum properties is system-relative. A variable q of system S may have a definite value relative to system O (because O has interacted with S) while remaining indefinite relative to system P (which has not interacted with S).

The formal content of RQM is expressed through the notion that every quantum mechanical description is indexed by a reference system: the state $|\psi\rangle_O$ denotes the state of S relative to O, and the "quantum event" of S taking a value $q = a$ is a relational fact involving both S and O.

3.2 What RQM Gets Right

From the perspective of Quantum Perspectivism, RQM correctly identifies several crucial features:

1. **Relationality is fundamental**: RQM correctly recognizes that quantum states are not absolute but observer-relative. This insight aligns precisely with the Yoneda Constraint's implication that physical properties are constituted by relations.
2. **No privileged observer**: RQM insists that any physical system can serve as a reference system, and there is no hierarchy among observers. This democratic treatment of perspectives corresponds to the universality of the Yoneda Lemma, which makes no distinction among objects.

3. **Dissolution of the measurement problem**: RQM's relational move dissolves the measurement problem by denying that there is a single, absolute quantum state that must "collapse." Instead, the definite value that O registers upon measuring S is simply the relational fact between O and S. This is structurally analogous to QP's treatment of measurement as presheaf restriction.

4. **Information as physical**: RQM emphasizes that the information one system has about another is the fundamental quantity. This information-theoretic emphasis is compatible with QP's treatment of presheaf sections as the fundamental physical data.

3.3 What RQM Is Missing

Despite its conceptual clarity, RQM suffers from several significant lacunae that Quantum Perspectivism fills:

1. **No mathematical framework for coherence**: RQM asserts that quantum states are relational but provides no mathematical mechanism ensuring that the various relational descriptions are *mutually consistent*. If O assigns state $|\psi\rangle_O$ to S and P assigns state $|\phi\rangle_P$ to S, what constrains the relationship between $|\psi\rangle_O$ and $|\phi\rangle_P$? RQM is silent on this point. QP answers it directly: the presheaf condition requires that for any refinement $f : C' \to C$, the restriction map $S(f)$ coherently relates the data at C and C'. The naturality of the presheaf is precisely the coherence condition that RQM lacks.

2. **No derivation of Hilbert space structure**: RQM takes the Hilbert space formalism as given and then interprets it relationally. It does not explain *why* quantum mechanics employs Hilbert spaces, complex amplitudes, or the Born rule. QP derives all of these from the Yoneda Constraint applied to a monoidal category of contexts.

3. **Vague ontology of "relations"**: While RQM insists that quantum events are relational, it does not provide a precise mathematical characterization of what a "relation" is in this context. The word "relation" in RQM is used informally, almost philosophically. In QP, a relation is a morphism in a category, and the Yoneda Lemma gives a precise theorem about how morphisms determine identity. The informal intuition of RQM is made rigorous.

4. **The problem of cross-perspective consistency**: Suppose O measures S and obtains result a, while P later measures the composite system $O+S$. RQM claims that P's measurement will find correlations consistent with O having obtained a, but the justification for this "consistency of cross-perspective facts" is never derived from first principles within RQM—it is essentially postulated. In QP, this consistency is a *theorem*: the naturality of the presheaf forces the commutativity

$$\begin{array}{ccc} S(C_{O+S}) & \xrightarrow{S(i_O)} & S(C_O) \\ {\scriptstyle S(f)}\downarrow & & \downarrow{\scriptstyle S(f')} \\ S(C'_{O+S}) & \xrightarrow[S(i'_O)]{} & S(C'_O) \end{array} \tag{4}$$

where C_O is the context associated with O's measurement, and the commutativity ensures that restricting to O's perspective commutes with further refinements.

5. **No account of quantum logic**: RQM does not address the non-Boolean logical structure of quantum propositions. QP derives quantum logic as the internal logic of the presheaf topos via its subobject classifier.

3.4 How the Yoneda Constraint Fills the Gaps

Proposition 3.1 (QP Subsumes RQM)**.** *Every claim of RQM is a consequence of QP. Specifically:*

(a) The RQM thesis that quantum states are observer-relative follows from the presheaf condition: the data $S(C)$ assigned by system S to context C is inherently relative to C.

(b) The RQM thesis that any system can be an observer follows from the universality of objects in $\mathscr{C}$: every context is an object, and the Yoneda Lemma applies to all objects without distinction.

(c) The RQM dissolution of the measurement problem follows from QP's treatment of measurement as presheaf restriction.

Moreover, QP extends RQM by:

(d) Providing the presheaf condition as a mathematical coherence constraint.

(e) Deriving the Hilbert space formalism from the monoidal structure of $\mathscr{C}$.

(f) Providing a topos-theoretic account of quantum logic.

(g) Grounding the relational ontology in a precise mathematical theorem (the Yoneda Lemma) rather than a philosophical postulate.

Remark 3.2. The relationship between QP and RQM is analogous to the relationship between general relativity and the equivalence principle. The equivalence principle is a physical insight—locally, gravity is indistinguishable from acceleration—but it does not by itself determine the mathematical structure of spacetime. General relativity provides the mathematical framework (Riemannian geometry, Einstein's equations) that realizes the equivalence principle in a complete, predictive theory. Similarly, RQM provides the physical insight—quantum states are relational—but does not determine the mathematical framework. QP provides the categorical framework (presheaf topos, Yoneda embedding) that realizes the relational insight in a mathematically complete structure.

3.5 Stable Facts and Cross-Perspective Consistency

Recent work by Rovelli, Di Biagio, and collaborators [48] has partially addressed the cross-perspective consistency problem by introducing the notion of "stable facts"—relational facts that, once established, remain stable under further interactions. A fact $q = a$ established between S and O is "stable" if any subsequent observer P who measures the $O + S$ composite will find results consistent with $q = a$.

This is an important development within RQM, but it remains an additional postulate rather than a derived result. In QP, the stability of relational facts is a *theorem*: the functoriality of the presheaf S guarantees that for any composable morphisms $f : C' \to C$ and $g : C'' \to C'$,

$$S(f \circ g) = S(g) \circ S(f), \tag{5}$$

which ensures that the data established at context C (via f) is preserved coherently when further refined to C'' (via g). The "stability" of RQM facts is thus a special case of presheaf functoriality—not an independent axiom but a mathematical consequence of the Yoneda Constraint.

Remark 3.3. The question of whether functoriality requires a physical mechanism or is a structural constraint is important. In QP, functoriality is a *structural constraint*: it is the defining property of a presheaf, not a dynamical law requiring a mechanism. Just as the transitivity of the identity relation requires no "mechanism," the functoriality of physical data requires no physical process—it is constitutive of what it means for data to be coherently relational.

3.6 Laudisa–Rovelli Stability Arguments and the Yoneda Response

Rovelli and Laudisa [3] have argued for the "stability" of the RQM interpretation under various philosophical challenges. One persistent objection is the "problem of the third party": if the state of S is relative to O, what prevents mutually contradictory relational facts? The RQM response appeals to the internal consistency of quantum mechanics—when P measures $O + S$, the results are always consistent.

QP strengthens this response decisively. The presheaf condition is not merely an appeal to "internal consistency" but a *functorial requirement*: for any commutative triangle of morphisms

$$\begin{tikzcd} C'' \arrow[r, "g"] \arrow[dr, "h"'] & C' \arrow[d, "f"] \\ & C \end{tikzcd} \tag{6}$$

in $\mathscr{C}$, we have $S(h) = S(g) \circ S(f)$. This is a strict mathematical constraint, not a hand-waving appeal to "consistency." The Yoneda Constraint thus provides RQM with the formal backbone it needs to resist objections based on cross-perspective consistency.

4 QBism: Quantum Bayesianism

4.1 The QBist Program

QBism (Quantum Bayesianism), developed by Fuchs, Mermin, Schack, and collaborators [4, 5, 7], represents a radical epistemic approach to quantum foundations. Its central tenets are:

(Q1) **Quantum states are personal judgments**: A quantum state ρ does not represent a physical property of a system but rather an agent's personal Bayesian credence about the outcomes of future measurements. Different agents may legitimately assign different quantum states to the same system.

(Q2) **Measurement outcomes are personal experiences**: The outcome of a measurement is a personal experience of the agent performing the measurement, not an objective fact about the world.

(Q3) **The Born rule is a normative constraint**: The Born rule is not a law of nature describing how probabilities arise from physical states; it is a *consistency requirement* (analogous to Dutch book coherence in Bayesian probability theory) that rational agents must satisfy.

(Q4) **Quantum mechanics is a "user's manual"**: QBism treats quantum mechanics as a tool that helps agents navigate their experiences—a kind of "user's manual for reality" rather than a "mirror of nature."

4.2 Points of Agreement

QP and QBism share several important commitments:

1. **No observer-independent quantum state**: Both QP and QBism reject the notion that a quantum state is an absolute, observer-independent property of a physical system. In QP, the quantum state is the data $S(C)$ relative to context C; in QBism, it is an agent's personal credence.
2. **Centrality of the agent/context**: Both frameworks give a central role to the "viewpoint" from which quantum descriptions are formulated. QBism calls this the "agent"; QP calls it the "observational context."
3. **The Born rule as a coherence condition**: QP derives the Born rule from the Yoneda isomorphism combined with Gleason's theorem, treating it as a structural consequence of perspectival consistency. QBism treats the Born rule as a normative coherence condition. Both thus view the Born rule as expressing consistency rather than describing a physical mechanism.
4. **Dissolution of the measurement problem**: Both frameworks avoid the measurement problem by denying that there is an objective, observer-independent wave function that must collapse.

4.3 The Fundamental Divergence: Epistemic vs. Ontic Perspectivism

Before identifying the divergence, we note that Fuchs has explicitly rejected the label "epistemic" for QBism, preferring the term "participatory realism" [6]. QBism's position is that quantum mechanics describes the *participatory* relationship between an agent and the world, and that this participation is ontologically fundamental—not merely a limitation on knowledge. We take this self-description seriously and frame the divergence accordingly: the issue is not "epistemic vs. ontic" in the crude sense, but rather the nature and locus of objectivity within a perspectival framework.

Despite the shared rejection of a "view from nowhere," QP and QBism diverge fundamentally on the question of *objectivity*.

Definition 4.1 (Epistemic Perspectivism)**.** A perspectival framework is **epistemic** if the perspectival nature of quantum states reflects limitations on what agents can *know*, while the underlying physical reality is (or may be) non-perspectival.

Definition 4.2 (Ontic Perspectivism)**.** A perspectival framework is **ontic** if the perspectival nature of quantum states reflects the actual structure of physical reality: reality itself is constituted by perspectival relations, and there is no non-perspectival "reality behind the appearances."

QBism is a form of epistemic perspectivism: quantum states reflect the agent's beliefs, but QBism remains deliberately agnostic about the nature of the underlying reality that gives rise to the agent's experiences. Fuchs has described QBism as "a philosophy of the experience of reality, not a philosophy of reality itself" [6].

QP, by contrast, is a form of **ontic perspectivism**. The presheaf $S : \mathscr{C}^{\mathrm{op}} \to \mathbf{Set}$ is an *objective mathematical entity*—it exists independently of any particular agent's beliefs. The perspectivalism lies not in the agent's epistemic limitations but in the *structure of reality itself*: physical reality is constituted by the coherent totality of perspectival data, and the Yoneda Lemma proves that this totality is complete. There is nothing "behind" the presheaf; the presheaf *is* the physical reality.

Proposition 4.3 (Objectivity of the Presheaf)**.** *In QP, the presheaf $S : \mathscr{C}^{\mathrm{op}} \to \mathbf{Set}$ is objective in the following precise senses:*

(a) S is a well-defined mathematical object in the topos $\widehat{\mathscr{C}}$, independent of any agent.

(b) The restriction maps $S(f) : S(C) \to S(C')$ are determined by the categorical structure, not by any agent's beliefs.

(c) Two systems S_1 and S_2 are physically identical if and only if the presheaves are naturally isomorphic: $S_1 \cong S_2$ in $\widehat{\mathscr{C}}$.

(d) The physical content of S is fully determined by the Yoneda Lemma—no reference to agents, beliefs, or subjective experience is needed.

4.4 The Problem of Solipsism and QBism's Response

A persistent criticism of QBism is the *charge of solipsism*: if quantum states are personal beliefs and measurement outcomes are personal experiences, does QBism collapse into a form of solipsism in which each agent is trapped in their own experiential bubble?

QBists respond that QBism is not solipsist because (i) it posits an external world that exists independently of any agent, and (ii) the Born rule provides a normative constraint that connects different agents' experiences. However, critics (e.g., [8, 9]) argue that this response is insufficient because QBism provides no account of *what* the external world is like, only of how agents should calibrate their expectations.

QP avoids the solipsism charge entirely. The presheaf S is an objective entity; different contexts C, C' access different aspects of this single objective entity. The coherence of cross-perspective data is not a matter of inter-subjective agreement between agents but a mathematical theorem (the presheaf condition). In QP, there is a "view from everywhere"—the presheaf itself—even though there is no "view from nowhere."

4.5 Technical Comparison: Credences vs. Presheaf Sections

Let us make the comparison technically precise. In QBism, an agent's quantum state assignment is a density operator ρ on a Hilbert space $\mathcal{H}$, which encodes the agent's credences about measurement outcomes via the Born rule:

$$p(k) = \mathrm{Tr}(\rho E_k), \tag{7}$$

where $\{E_k\}$ is a POVM representing the measurement. The density operator ρ is "personal"—it encodes one agent's beliefs and may differ from another agent's.

In QP, the corresponding structure is a section $s \in S(C)$ of the presheaf at context C. The restriction map $S(f) : S(C) \to S(C')$ provides the objective transformation of data under change of context. The Born rule probability emerges as:

$$p(\lambda|C) = |\langle e_\lambda, S(f)(\psi)\rangle|^2, \tag{8}$$

where the inner product is the perspectival inner product derived from the coherence conditions.

The critical difference: in QBism, ρ is a subjective input; in QP, S and its restrictions are *derived* from the categorical structure. There is no freedom for different agents to disagree about S itself—they may occupy different contexts C, C' and therefore have access to different data $S(C), S(C')$, but these are objective and deterministically related by the restriction maps.

4.6 The Dutch Book vs. the Yoneda Lemma

QBism motivates the Born rule as a "quantum Dutch book" coherence condition [7]: an agent who violates the Born rule can be made to accept a series of bets that guarantee a loss. This is a normative argument—it tells agents how they *should* assign probabilities.

QP provides a *structural* derivation: the Born rule is forced by the Yoneda isomorphism combined with the positivity and normalization of probability measures on the subobject lattice (Gleason's theorem in the presheaf topos). This is not normative but constitutive—the Born rule is not a rule of rational behavior but a consequence of the mathematical structure of perspectival physics.

Remark 4.4. The QP derivation of the Born rule is arguably more satisfying because it explains *why* the Born rule takes the specific form $p = |\langle e_\lambda, \psi\rangle|^2$ rather than any other functional form. QBism's Dutch book argument shows that the Born rule is *consistent* but does not explain why it is *necessary*. QP's derivation shows that it is the unique probability assignment compatible with the Yoneda isomorphism.

5 The Many-Worlds Interpretation

5.1 Everett's Original Vision

Hugh Everett III's "relative state" formulation [10], later developed into the Many-Worlds Interpretation (MWI) by DeWitt and others [11], represents the most ambitious attempt to take the quantum formalism at face value. Its core commitment is:

(E1) **Universal wave function**: There exists a universal quantum state $|\Psi\rangle$ of the entire universe, evolving unitarily at all times.

(E2) **No collapse**: The projection postulate is denied. Measurements do not cause wave function collapse; instead, the universal state develops into a superposition of branches, each representing a definite measurement outcome.

(E3) **All branches are real**: Every branch of the universal wave function is equally real. The appearance of a single definite outcome is an illusion arising from the perspective of observers within a single branch.

5.2 Branches as Context-Restrictions

From the QP perspective, the Everettian branches have a natural interpretation: they are **context-restrictions of the presheaf**.

Let $S : \mathscr{C}^{\mathrm{op}} \to \mathbf{Set}$ be the presheaf representing the physical universe. A "branch" corresponds to the data $S(C)$ at a particular context C—the restriction of the universal presheaf to a specific observational perspective. The "branching" event is the process of restricting from a coarser context C (in which the system is in superposition) to a finer context C' (in which the system has a definite value):

$$S(f) : S(C) \to S(C'), \quad f : C' \to C. \tag{9}$$

Proposition 5.1 (MWI Branches as Presheaf Restrictions)**.** *The set of Everettian branches at a given measurement event is isomorphic to the set of fibers of the restriction map $S(f)$ at the relevant measurement morphism $f : C_{meas} \to C_{pre}$. Specifically, if α is the measured observable with eigenvalues $\{\lambda_i\}$, then:*

$$\{\textit{branches}\} \cong \{S(C_{\lambda_i})\}_i, \tag{10}$$

where C_{λ_i} is the context in which α has the definite value λ_i.

5.3 The Advantage: No Extravagant Multiverse

The MWI pays a steep ontological price for preserving unitarity: it posits an exponentially branching multiverse of equally real worlds. This ontological extravagance has been criticized by many authors (e.g., [15, 16]).

QP achieves the same formal benefits—universal unitarity, no collapse—without the ontological cost. In QP, the "branches" are not separate worlds but *different perspectives on the same presheaf.* The presheaf is the single, complete physical reality; different contexts access different aspects of it. There is no need to posit the independent existence of the other branches as separate physical worlds. They exist as potential perspectives—mathematical sections of the presheaf that can in principle be accessed by appropriate morphisms—but they do not constitute separate, causally disconnected worlds.

Remark 5.2 (Ontological Economy). QP satisfies a principle of ontological economy that the MWI violates. The MWI multiplies worlds; QP multiplies perspectives. But perspectives are not additional ontological entities—they are aspects of a single entity (the presheaf), just as the different views of a mountain are not additional mountains.

5.4 The Preferred Basis Problem Resolved

One of the most serious technical problems facing the MWI is the **preferred basis problem**: in what basis does the universal wave function "branch"? The wave function is a vector in Hilbert space, and Hilbert space has no preferred basis. Different choices of basis lead to different "branch structures," and the MWI provides no mechanism for selecting one.

Decoherence theory [17, 18] is often invoked to solve this problem: the interaction between a system and its environment selects a "pointer basis" in which the reduced density matrix of the system becomes approximately diagonal. However, decoherence is a *quantitative* matter (it is never perfectly complete), and it is not clear that it provides a *principled* solution to what is fundamentally a structural problem.

QP provides a clean resolution. The "basis" in which branching occurs is not a feature of the Hilbert space but of the **category of contexts**. Different measurement contexts C_α (corresponding to measurement of observable α) and C_β (corresponding to measurement of observable β) define different decompositions of the presheaf data. The "preferred basis" for a given measurement event is simply the context that is actually realized—the morphism that is actually selected.

Proposition 5.3 (Resolution of the Preferred Basis Problem)**.** *In QP, the question "in what basis does the universe branch?" is replaced by the well-posed question "what context is selected by the morphism $f : C_{lab} \to C_{meas}$?" The answer is determined by the physical interaction between the system and the measurement apparatus, which corresponds to a specific morphism in $\mathscr{C}$.*

5.5 Probability in QP vs. Many-Worlds

The **probability problem** is widely regarded as the most serious challenge to the MWI. If all branches are equally real, why should we assign Born rule probabilities to outcomes? Why not assign equal probability to each branch, regardless of the amplitude?

Various solutions have been proposed. The "decision-theoretic" approach of Deutsch [13] and Wallace [12] argues that a rational agent in a branching universe should adopt Born rule probabilities. The "self-locating uncertainty" approach of Vaidman [14] and others argues that an agent who is about to branch should be uncertain about which branch they will end up in.

QP avoids this problem entirely because branches are not separate worlds with independent existence. The Born rule in QP is a theorem—a consequence of the

Yoneda isomorphism and Gleason's theorem applied within the presheaf topos (Theorem 3.3 of [1]). There is no need for a separate justification of why probabilities take the Born rule form, because the Born rule is not an additional postulate but a derived consequence of the categorical structure.

5.6 Everett's "Relative State" and QP's Presheaf Restriction

It is worth noting that Everett's *original* formulation—the "relative state" formulation—is closer to QP than the later "many-worlds" gloss. Everett defined the state of a subsystem S relative to a particular state of the observer O, and showed that this relative state evolves consistently with the overall unitary evolution. This is essentially the presheaf restriction: the state of S at the context defined by O's state is the restriction $S(C_O)$.

The deviation occurred when DeWitt reinterpreted Everett's relative states as "equally real worlds." QP can be seen as a return to the spirit of Everett's original insight, now equipped with the mathematical framework (the presheaf topos) that Everett lacked.

6 Topos Quantum Theory

6.1 The Isham–Butterfield Program

The topos-theoretic approach to quantum mechanics was initiated by Isham and Butterfield in a landmark series of papers [19, 20, 21, 22] and further developed by Isham and Döring [23, 24, 25]. This program represents the most sophisticated mathematical attempt to reformulate quantum mechanics in terms of presheaves and topoi prior to Quantum Perspectivism.

The core idea of the Isham–Butterfield–Döring (IBD) program is to reformulate quantum mechanics not in the topos **Set** (where standard quantum mechanics lives) but in a presheaf topos $\widehat{\mathcal{V}(\mathcal{H})}$, where $\mathcal{V}(\mathcal{H})$ is the *context category*: the poset of abelian von Neumann subalgebras of the algebra $\mathcal{B}(\mathcal{H})$ of bounded operators on a Hilbert space $\mathcal{H}$, ordered by inclusion.

Definition 6.1 (Context Category in IBD)**.** The **context category** $\mathcal{V}(\mathcal{H})$ is the poset (viewed as a category) whose objects are abelian von Neumann subalgebras $V \subseteq \mathcal{B}(\mathcal{H})$ and whose morphisms are inclusions $V' \hookrightarrow V$ (note the direction: morphisms go from smaller to larger algebras in the original formulation, or equivalently, the poset is ordered by reverse inclusion for the presheaf formulation).

6.2 Daseinisation and the Spectral Presheaf

The central technical construction of the IBD program is **daseinisation**—a process that approximates a quantum proposition (a projection operator $\hat{P}$) by the "best available" classical proposition at each context V.

Definition 6.2 (Outer Daseinisation)**.** Given a projection $\hat{P} \in \mathcal{B}(\mathcal{H})$ and a context $V \in \mathcal{V}(\mathcal{H})$, the **outer daseinisation** of $\hat{P}$ at V is:

$$\delta^o(\hat{P})_V = \bigwedge\{\hat{Q} \in \mathcal{P}(V) : \hat{Q} \geq \hat{P}\}, \tag{11}$$

where $\mathcal{P}(V)$ denotes the projection lattice of V and $\bigwedge$ denotes the greatest lower bound (which equals the infimum in this case).

The **spectral presheaf** $\underline{\Sigma}$ assigns to each context V the Gel'fand spectrum Σ_V of V—the space of multiplicative linear functionals on V, which represents the "classical state space" at context V.

Definition 6.3 (Spectral Presheaf)**.** The **spectral presheaf** $\underline{\Sigma} : \mathcal{V}(\mathcal{H})^{\mathrm{op}} \to \mathbf{Set}$ assigns:

- To each context V: the Gel'fand spectrum Σ_V of V.
- To each inclusion $i_{V'V} : V' \hookrightarrow V$: the restriction map $\underline{\Sigma}(i_{V'V}) : \Sigma_V \to \Sigma_{V'}$ given by restricting multiplicative linear functionals from V to V'.

6.3 The Kochen–Specker Theorem, Categorically

One of the most beautiful results of the IBD program is the categorical reformulation of the Kochen–Specker theorem [26]:

Theorem 6.4 (Kochen–Specker via Presheaves)**.** *The spectral presheaf $\underline{\Sigma}$ has no global sections. That is, there exists no natural transformation $\mathbf{1} \to \underline{\Sigma}$ from the terminal presheaf $\mathbf{1}$ to $\underline{\Sigma}$ (for $\dim \mathcal{H} \geq 3$).*

This is a presheaf-theoretic reformulation of the Kochen–Specker theorem: the impossibility of assigning simultaneous definite values to all observables is equivalent to the non-existence of a global section of the spectral presheaf.

In QP, this result acquires a deeper significance: the absence of global sections is not a mysterious "no-go" result but a direct expression of the fact that the presheaf topos has a non-Boolean subobject classifier. Physical reality is perspectival, and the failure of global sections is simply the mathematical expression of this perspectivalism.

6.4 QP as the Physical Motivation for IBD

The IBD program has been criticized for being mathematically sophisticated but physically unmotivated. Why should we reformulate quantum mechanics in a presheaf topos? The IBD papers offer technical reasons (e.g., recovering a form of realism compatible with quantum mechanics) but no deep physical principle that *forces* the topos-theoretic formulation.

QP provides this missing physical motivation: the Yoneda Constraint. The reason quantum mechanics naturally lives in a presheaf topos is that the Yoneda Lemma requires physical systems to be presheaves—coherent assignments of data to observational contexts. The topos $\widehat{\mathscr{C}}$ is not an arbitrary mathematical choice but the *canonical setting* for Yoneda-constrained physics.

Proposition 6.5 (QP Motivates IBD)**.** *The IBD program's choice of the presheaf topos $\widehat{\mathcal{V}(\mathcal{H})}$ as the arena for quantum physics is a consequence of the Yoneda Constraint applied to the context category $\mathcal{V}(\mathcal{H})$. Specifically:*

(a) The context category $\mathcal{V}(\mathcal{H})$ is a special case of the QP context category $\mathscr{C}$, where contexts are identified with abelian subalgebras (classical perspectives on the quantum system).

(b) *The spectral presheaf $\underline{\Sigma}$ is the QP presheaf S restricted to the context category $\mathcal{V}(\mathcal{H})$.*

(c) *Daseinisation is the QP mechanism of presheaf restriction: approximating quantum data by its "best classical approximation" at each context.*

6.5 Beyond IBD: What QP Adds

While QP motivates the IBD program, it also extends it in several important directions:

1. **Derivation of Hilbert space structure**: IBD takes the Hilbert space $\mathcal{H}$ and the algebra $\mathcal{B}(\mathcal{H})$ as given, then constructs the presheaf topos over the context category derived from $\mathcal{B}(\mathcal{H})$. QP derives the Hilbert space structure itself from the Yoneda Constraint, making the IBD construction a *consequence* rather than a starting point.

2. **General context categories**: IBD restricts to the specific context category $\mathcal{V}(\mathcal{H})$. QP works with a general category of observational contexts $\mathscr{C}$, which may include contexts not naturally identified with abelian subalgebras (e.g., spacetime regions, as in AQFT, or abstract operational contexts).

3. **Dynamics**: The IBD program has struggled to incorporate dynamics satisfactorily. QP provides a natural treatment of dynamics as one-parameter families of natural automorphisms of the presheaf, yielding the Schrödinger equation via Stone's theorem.

4. **The Born rule**: IBD does not derive the Born rule from the topos structure; it must be added as an additional ingredient. QP derives the Born rule from the Yoneda isomorphism combined with Gleason's theorem in the presheaf topos.

6.6 The Heunen–Landsman–Spitters "Bohrification" Program

A complementary topos approach was developed by Heunen, Landsman, and Spitters [49] under the name "Bohrification." While the IBD program uses the *contravariant* (presheaf) approach with outer daseinisation, the Bohrification program uses a *covariant* approach: the context category $\mathcal{V}(\mathcal{H})$ is ordered by *inclusion* (rather than reverse inclusion), and the relevant topos is the topos of *covariant* functors $\mathcal{V}(\mathcal{H}) \to \mathbf{Set}$.

In the Bohrification framework, each context V is viewed as a "classical snapshot" of the quantum system (following Bohr's dictum that measurement results must be expressible in classical terms), and the system as a whole is represented by a commutative C*-algebra internal to the topos—the "Bohrification" of the original noncommutative algebra $\mathcal{B}(\mathcal{H})$.

QP accommodates both the contravariant (IBD) and covariant (Bohrification) approaches as special cases of the general presheaf construction over $\mathscr{C}$. The choice between contravariant and covariant formulations corresponds to a choice of variance for the presheaf—whether morphisms in $\mathscr{C}$ represent "refinements" (contravariant, as in QP and IBD) or "coarsenings" (covariant, as in Bohrification). The Yoneda Constraint applies equally to both, since the Yoneda Lemma holds for presheaves on $\mathscr{C}^{\mathrm{op}}$ as well as $\mathscr{C}$.

6.7 Döring–Isham "Topos Physics"

In their later work [23, 25], Döring and Isham proposed a broader program of "topos physics" in which any physical theory is formulated within a topos, with the specific topos depending on the theory. They introduced the notion of a "neo-realist" interpretation: truth values in the topos-theoretic formulation are elements of the subobject classifier Ω (which is multi-valued), and physical propositions are "contextually true" or "contextually false" at each context.

This neo-realist program aligns closely with QP's ontic structural realism. The multi-valued truth values of the presheaf topos are precisely the perspectival truth values of QP: a proposition may be "true from context C" and "false from context C'," and this is not a deficiency of the formalism but a faithful representation of perspectival physical reality.

Proposition 6.6 (QP Truth Values)**.** *In QP, the truth value of a physical proposition about system S is an element of the subobject classifier Ω of the presheaf topos $\widehat{\mathscr{C}}$. For context C, $\Omega(C)$ is the set of sieves on C—collections of morphisms into C closed under precomposition. A proposition is "globally true" only if the corresponding sieve is the maximal sieve (the sieve containing all morphisms into C). The Kochen–Specker theorem is the statement that not all physically meaningful propositions can be simultaneously globally true.*

7 Categorical Quantum Mechanics (Abramsky–Coecke)

7.1 The CQM Program

The Categorical Quantum Mechanics (CQM) program, initiated by Abramsky and Coecke [27] and extensively developed by Coecke, Kissinger, and collaborators [28, 29, 30], takes a different categorical approach to quantum foundations. Rather than working with presheaves and topoi, CQM works within **compact closed categories** (and their refinements: dagger compact categories, dagger symmetric monoidal categories).

Definition 7.1 (Compact Closed Category)**.** A **symmetric monoidal category** $(\mathscr{C}, \otimes, I)$ is **compact closed** if every object A has a dual A^* equipped with morphisms

$$\eta_A : I \to A \otimes A^*, \qquad \varepsilon_A : A^* \otimes A \to I \tag{12}$$

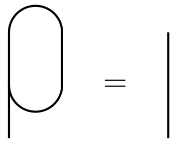
satisfying the "snake equations" (zigzag identities):

$$(\varepsilon_A \otimes \mathrm{id}_A) \circ (\mathrm{id}_A \otimes \eta_A) = \mathrm{id}_A, \qquad (\mathrm{id}_{A^*} \otimes \varepsilon_A) \circ (\eta_A \otimes \mathrm{id}_{A^*}) = \mathrm{id}_{A^*}. \tag{13}$$

The paradigmatic example is **FdHilb**, the category of finite-dimensional Hilbert spaces and linear maps, which is compact closed with $A^* = \bar{A}$ (the conjugate space).

7.2 String Diagrams and Graphical Calculus

A major innovation of CQM is the use of **string diagrams**—a graphical calculus in which morphisms are represented by boxes, objects by wires, and composition/tensor product by vertical/horizontal concatenation. The snake equations become "straightening" of bent wires:



String diagrams have proven remarkably powerful for reasoning about quantum protocols (teleportation, entanglement swapping, quantum error correction) and have led to new algorithms for classical simulation of quantum circuits [28].

7.3 Complementarity in CQM: Frobenius Algebras

CQM formalizes complementary observables using **special commutative Frobenius algebras** (SCFAs) in a dagger symmetric monoidal category. An observable with n outcomes is represented by a copying morphism $\delta : A \to A \otimes A$ and a deleting morphism $\epsilon : A \to I$ satisfying the Frobenius and specialness axioms:

$$(\delta \otimes \mathrm{id}) \circ \delta = (\mathrm{id} \otimes \delta) \circ \delta, \qquad \delta^{\dagger} \circ \delta = \mathrm{id}_A. \tag{14}$$

Two observables are **complementary** if their corresponding Frobenius algebras satisfy certain algebraic conditions (the "complementarity equations" of Coecke and Duncan [29]).

7.4 CQM and QP: Complementary Programs

The relationship between CQM and QP is one of **complementarity rather than competition**. The two programs operate at different categorical levels and address different questions:

Feature	**CQM**	**QP**
Starting point	**Hilb** as given	**Hilb** derived
Categorical tool	Compact closed categories	Presheaf topoi
Central question	What can we compute?	Why quantum?
Mathematical emphasis	Monoidal structure, duality	Yoneda Lemma, presheaves
Observables	Frobenius algebras	Natural transformations
Physical content	Compositional, operational	Foundational, ontological

1. **CQM takes Hilb as given; QP derives it**: CQM begins with the category **FdHilb** (or a suitable abstract category satisfying the axioms of a dagger compact category) and explores its compositional structure. QP explains *why* the category of physical systems has this structure by deriving it from the Yoneda Constraint. CQM is thus a "downstream" program: once QP has established that quantum mechanics is the unique physics consistent with the Yoneda Constraint, CQM provides the compositional calculus for reasoning within the resulting framework.

2. **Different categorical levels**: CQM works *within* a single category (typically **FdHilb** or an abstraction thereof). QP works with the *presheaf category over* a base category of contexts. The relationship is akin to the relationship between algebra (working within a ring) and algebraic geometry (studying the presheaves on a topological space).

3. **Synthesis**: The two programs can be synthesized as follows. The Yoneda Constraint forces the fibers $S(C)$ to be Hilbert spaces (as derived in [1]). The compositional structure of these Hilbert spaces—their monoidal products, dualities, and diagrammatic calculus—is then described by CQM. QP provides the "why"; CQM provides the "how."

Proposition 7.2 (QP–CQM Synthesis)**.** *Let $\mathscr{C}$ be the QP context category and $S : \mathscr{C}^{\mathrm{op}} \to$ **Hilb** the presheaf with Hilbert space fibers (as derived from the Yoneda Constraint). The category* **Hilb** *carries a compact closed structure, and the compositional tools of CQM apply to the fibers of S. Specifically:*

(a) The tensor product $S(C) \otimes S(C')$ encodes the CQM compositional structure of composite systems.

(b) Complementary observables in the sense of CQM (pairs of SCFAs satisfying complementarity equations) correspond to non-commutative contexts C_α, C_β in QP.

(c) The string-diagrammatic calculus of CQM provides an efficient computational tool for working with the fibers of the QP presheaf.

7.5 ZX-Calculus and the Perspectival Structure

The ZX-calculus [29, 31] is a refinement of the CQM string-diagrammatic approach specifically tailored to qubit quantum computation. It uses "Z-spiders" and "X-spiders" (representing measurement in the computational and Hadamard bases, respectively) connected by wires.

From the QP perspective, the two types of spiders correspond to two specific contexts C_Z and C_X in $\mathscr{C}$, and the ZX-calculus rules are the restriction maps and their compositions in the presheaf. The "complementarity" of Z and X in the ZX-calculus is precisely the non-commutativity of the contexts C_Z and C_X in QP.

8 Consistent and Decoherent Histories

8.1 The Consistent Histories Framework

The consistent (or decoherent) histories approach to quantum mechanics, developed by Griffiths [32, 33], Omnès [34], and Gell-Mann and Hartle [35], provides a framework for assigning probabilities to sequences of quantum events ("histories") without invoking measurement or wave function collapse.

A **history** is a time-ordered sequence of projection operators:

$$\mathcal{Y} = (P^1_{a_1}(t_1), P^2_{a_2}(t_2), \ldots, P^n_{a_n}(t_n)), \tag{15}$$

where $P^k_{a_k}(t_k)$ is a projection onto the eigenspace of observable α_k with eigenvalue a_k at time t_k.

A set of histories is **consistent** (or **decoherent**) if the decoherence functional

$$D(\mathcal{Y}, \mathcal{Y}') = \mathrm{Tr}\left(C_{\mathcal{Y}} \rho C^\dagger_{\mathcal{Y}'}\right) \tag{16}$$

satisfies $D(\mathcal{Y}, \mathcal{Y}') = 0$ for $\mathcal{Y} \neq \mathcal{Y}'$, where $C_{\mathcal{Y}} = P^n_{a_n}(t_n) \cdots P^1_{a_1}(t_1)$ is the **class operator**. When this condition is satisfied, the diagonal elements $D(\mathcal{Y}, \mathcal{Y})$ yield a well-defined probability distribution over histories.

8.2 Histories as Presheaf Sections over Temporal Contexts

In QP, the consistent histories framework can be naturally embedded by enriching the context category $\mathscr{C}$ to include **temporal structure**.

Definition 8.1 (Temporal Context Category)**.** Let $\mathscr{C}_T$ be the category whose objects are **temporal contexts** (C, t)—an observational context C at time t—and whose morphisms include both:

- **Spatial refinements**: $f : (C', t) \to (C, t)$ at fixed time (the same as in the original $\mathscr{C}$).
- **Temporal morphisms**: $g : (C, t') \to (C, t)$ for $t' > t$, representing the passage of time within a context.

A history in the Griffiths sense is then a particular kind of section of the presheaf $S : \mathscr{C}_T^{\mathrm{op}} \to \mathbf{Set}$—one that specifies the data at a sequence of temporal contexts.

Proposition 8.2 (Consistent Histories as Global Sections)**.** *A consistent set of histories for a system S corresponds to a collection of sections of the presheaf S over a chain of temporal contexts $(C_1, t_1) \to (C_2, t_2) \to \cdots \to (C_n, t_n)$ that are mutually compatible in the sense that their overlaps (via the decoherence functional) vanish. The consistency condition is a consequence of the presheaf condition: the restriction maps along the temporal morphisms must satisfy the functoriality axiom $S(g \circ f) = S(f) \circ S(g)$.*

8.3 The Problem of Multiple Consistent Sets

A well-known problem with the consistent histories approach is the **problem of multiple consistent sets**: for a given quantum system, there are many different sets of histories that are individually consistent but mutually incompatible. Different consistent sets can assign contradictory truth values to the same proposition, and there is no principle within the consistent histories framework for selecting one set over another.

QP resolves this problem by recognizing that different consistent sets correspond to *different contexts* (different chains in $\mathscr{C}_T$). The "contradictions" between different consistent sets are not genuine contradictions but perspectival differences: a proposition that is true at context C may be false at context C', and this is the expected behavior of a presheaf with non-Boolean logic. The multi-valued subobject classifier Ω of the presheaf topos provides the logical framework for handling these "contradictions" without inconsistency.

8.4 Gell-Mann–Hartle and the Decoherence Functional

Gell-Mann and Hartle [35] extended the consistent histories approach to quantum cosmology, where there is no external observer to perform measurements. Their decoherent histories program treats the decoherence functional as fundamental, with classical behavior emerging when the decoherence is sufficiently strong.

In QP, the decoherence functional has a natural categorical interpretation: it measures the "overlap" between different presheaf sections, analogous to the inner product between different states. The decoherence condition $D(\mathcal{Y}, \mathcal{Y}') = 0$ for $\mathcal{Y} \neq \mathcal{Y}'$ corresponds to the orthogonality of the presheaf sections associated with different histories. Decoherence—the process by which quantum coherence is suppressed—is the categorical operation of coarse-graining: restricting the presheaf from a fine-grained context category to a coarser subcategory, as detailed in Section 8.3 of [1].

Proposition 8.3 (Decoherence as Coarse-Graining)**.** *Let $i : \mathscr{C}_{coarse} \hookrightarrow \mathscr{C}$ be the inclusion of a coarse-grained subcategory. The restriction $i^*S = S \circ i^{\mathrm{op}}$ "decoheres" the presheaf: off-diagonal coherences visible in $\mathscr{C}$ become invisible in $\mathscr{C}_{coarse}$. The decoherence functional of Gell-Mann and Hartle measures the failure of i^* to preserve these off-diagonal terms.*

9 Other Interpretive Frameworks

9.1 Objective Collapse Theories: GRW and Penrose

9.1.1 The GRW Theory

The Ghirardi–Rimini–Weber (GRW) theory [36] is the most developed **objective collapse theory**: it modifies the Schrödinger equation by adding a stochastic, nonlinear collapse mechanism that operates spontaneously, without requiring measurement. Each elementary particle has a small probability per unit time ($\lambda \approx 10^{-16}\,\mathrm{s}^{-1}$) of undergoing a spontaneous localization—a "hit"—that collapses the wave function to a narrow Gaussian. For individual particles, this happens so rarely that quantum behavior is preserved; for macroscopic objects (with $\sim 10^{23}$ particles), the collapse rate scales up enormously, ensuring rapid localization.

From the QP perspective, the GRW modification represents a *category error*: it treats the presheaf restriction (the transition from superposition to definite value) as a physical process requiring a dynamical mechanism. In QP, definiteness is not achieved by a physical collapse but by the selection of a context—a morphism in $\mathscr{C}$. The "collapse" is the mathematical operation of restricting the presheaf, not a physical event.

Remark 9.1. The GRW theory is empirically distinguishable from standard quantum mechanics and from QP: it predicts deviations from unitary evolution for mesoscopic systems, a small but nonzero rate of energy non-conservation, and a universal noise background. Current experimental bounds constrain but do not rule out GRW [37]. If future experiments were to confirm GRW-type deviations, this would refute not only QP but standard quantum mechanics itself, suggesting that the Yoneda Constraint admits exceptions at some scale.

9.1.2 Penrose's Gravitational Collapse

Roger Penrose [38, 39] has proposed that wave function collapse is a real physical process driven by gravity: when a quantum superposition involves two states with significantly different mass-energy distributions, the gravitational self-energy of the difference creates an instability that triggers collapse. The collapse time is estimated as $\tau \sim \hbar/\Delta E_G$, where ΔE_G is the gravitational self-energy of the superposition.

QP provides a different perspective on the gravity-quantum interface. Rather than gravity *causing* collapse, QP suggests that gravity *is* a feature of the categorical structure of contexts: the Grothendieck topology on $\mathscr{C}$ encodes geometric information, and Einstein's equations emerge as constraints on this topology (Section 6 of [1]). The "collapse" in QP is always context-restriction, and the role of gravity is to shape the space of available contexts, not to trigger a collapse mechanism.

Proposition 9.2 (QP vs. Gravitational Collapse)**.** *In QP, the Penrose threshold $\tau \sim \hbar/\Delta E_G$ has a reinterpretation: it estimates the timescale on which the gravitational structure of spacetime (encoded in the Grothendieck topology of $\mathscr{C}$) forces a natural decoherence—a coarse-graining from fine to coarser contexts—that mimics the appearance of collapse. This decoherence is an objective feature of the categorical structure, not a modification of quantum mechanics.*

9.2 De Broglie–Bohm Pilot Wave Theory

The de Broglie–Bohm theory [40, 41] (also known as Bohmian mechanics) is the most developed **hidden variable theory**. It supplements the wave function $\psi(x,t)$ with actual particle positions $Q(t) = (Q_1(t), \ldots, Q_N(t))$ that evolve deterministically according to the guidance equation:

$$\frac{dQ_k}{dt} = \frac{\hbar}{m_k}\mathrm{Im}\left(\frac{\nabla_k\psi}{\psi}\right)\bigg|_{x=Q(t)}. \tag{17}$$

The particle positions are "hidden variables" in the sense that they are not determined by the wave function alone but require initial conditions. The Born rule is recovered as the equilibrium distribution: if the initial positions are distributed according to $|\psi|^2$, they remain so distributed for all time ("equivariance").

9.2.1 QP's Verdict on Hidden Variables

The Yoneda Constraint rules out hidden variables of the Bohmian type:

Theorem 9.3 (No Hidden Variables from Yoneda)**.** *Let $S : \mathscr{C}^{\mathrm{op}} \to \mathbf{Set}$ be a physical system satisfying the Yoneda Constraint. Then S admits no "hidden" properties—properties not determined by the relational profile $\mathsf{y}(S)$. Specifically, if two states $s_1, s_2 \in S(C)$ are indistinguishable at all contexts (i.e., for every morphism $f : C' \to C$ in $\mathscr{C}$, $S(f)(s_1) = S(f)(s_2)$), then $s_1 = s_2$.*

Proof. This follows directly from the full faithfulness of the Yoneda embedding. If s_1 and s_2 are indistinguishable at all contexts, then they determine the same natural transformation $\mathsf{y}(C) \to S$ via the Yoneda isomorphism $\mathrm{Nat}(\mathsf{y}(C), S) \cong S(C)$, hence $s_1 = s_2$. □

The Bohmian particle positions are precisely the kind of hidden property that the Yoneda Constraint excludes: they are variables that supplement the relational profile (the wave function) with additional information (the actual positions). In QP, there is simply no room for such supplementary structure.

9.2.2 The Non-Contextuality Loophole

One might object that Bell's and Kochen–Specker's no-go theorems rule out only *non-contextual* hidden variables, and Bohmian mechanics is explicitly *contextual*. Does the Yoneda Constraint similarly only rule out non-contextual hidden variables?

The answer is no: the Yoneda Constraint rules out hidden variables entirely, contextual or not. The key is that in QP, "context" is already built into the framework (it is the base category $\mathscr{C}$), and the presheaf already encodes all context-dependent data. There is no additional room for context-dependent hidden variables because the presheaf *is* the complete context-dependent description.

9.3 The Transactional Interpretation

The transactional interpretation (TI), proposed by Cramer [42] and recently developed by Kastner [43], describes quantum processes as "transactions" between emitters and absorbers, mediated by offer waves (advanced waves traveling backward in time) and confirmation waves (retarded waves traveling forward).

From the QP standpoint, the transactional interpretation can be partially recast in categorical terms:

1. The "offer wave" from emitter to absorber corresponds to a morphism $f : C_{\text{absorber}} \to C_{\text{emitter}}$ in $\mathscr{C}$.
2. The "confirmation wave" from absorber back to emitter corresponds to the opposite morphism in $\mathscr{C}^{\text{op}}$, i.e., the action of the presheaf on f^{op}.
3. The "transaction" (the completed quantum event) corresponds to the presheaf restriction along f: the data $S(f) : S(C_{\text{emitter}}) \to S(C_{\text{absorber}})$.

However, the TI's reliance on time-symmetric processes (advanced waves) is not needed in QP. The directionality of morphisms in $\mathscr{C}$ already encodes the relevant temporal structure, and the functoriality of the presheaf ensures consistency without requiring backward-in-time propagation.

9.4 Modal Interpretations

Modal interpretations of quantum mechanics [44, 45, 46] attempt to assign definite values to some (but not all) observables at all times, subject to the constraint that the assigned values are consistent with the quantum formalism. The "preferred observable" that receives a definite value is typically selected by the biorthogonal decomposition of the system-apparatus state.

In QP, modal interpretations correspond to selecting a particular subpresheaf of the full presheaf S—one that assigns definite values to some observables while leaving others indefinite. The biorthogonal decomposition of the modal interpretation is the categorical analogue of the Schmidt decomposition in the product context category.

The advantage of QP over modal interpretations is that QP does not need to select a preferred observable: the "definiteness" of a value is always relative to a context, and different contexts may make different observables definite. The modal interpretation's struggle with the "preferred basis problem" and the "loophole problem" [47] does not arise in QP because definiteness is contextual by construction.

10 Systematic Comparison Table

We now provide a comprehensive comparison of all frameworks discussed, organized along several dimensions of analysis.

Dimension	**QP**	**RQM**	**QBism**	**Many-Worlds**	**Topos (IBD)**
Ontology	Ontic structural realism	Relational	Agnostic (experiential)	Universal realism	Neo-realist
ψ status	Presheaf section (objective)	Relative to observer	Personal credence	Real universal state	Presheaf section
Measurement	Context restriction	Relational event	Personal experience	Branching	Daseinisation
Collapse	None (restriction)	Perspectival (apparent)	N/A (belief update)	None (branching)	None (approximation)
Born rule	Derived (Yoneda + Gleason)	Postulated	Normative (Dutch book)	Problematic	Not derived
Hidden variables	Excluded (Yoneda)	Excluded (completeness)	N/A	Excluded	Excluded (KS)
Quantum logic	Topos logic derived	Not addressed	N/A	Classical within branches	Topos logic
Math framework	Presheaf topos	Standard QM	Standard QM	Standard QM	Presheaf topos
Hilb status	Derived	Assumed	Assumed	Assumed	Assumed
Solipsism risk	None	Low	Moderate	None	None
Ontological cost	Minimal	Minimal	N/A	Very high	Moderate
Dynamics	Natural automorphisms	Standard	Standard	Standard Schrödinger	Problematic
Preferred basis	Resolved (contexts)	N/A	N/A	Problematic	N/A
Multi-observer consistency	Theorem (naturality)	Postulated	Inter-subjective	Trivial (branching)	Functoriality

Dimension	CQM	Consistent Histories	GRW	Bohm	Trans-actional
Ontology	Compositional/ operational	Multiple consistent sets	Ontological wave	Particles + wave	Emitters + absorbers
ψ status	Morphism in category	Framework for histories	Real, collapses	Real, guides	Offer wave
Measurement	Protocol in category	Decoherent history selection	Spontaneous collapse	Revealing position	Completed transaction
Collapse	N/A (compositional)	Decoherence	Physical (GRW hits)	None (effective)	Transaction completion
Born rule	Assumed (in **Hilb**)	Derived (within consistent sets)	Modified (collapse dynamics)	Equivariance (equilibrium)	Derived (transaction)
Hidden variables	N/A	N/A	None	Yes (positions)	None
Quantum logic	N/A	Classical within sets	N/A	Classical (contextual)	N/A
Math framework	Compact closed cat.	Projection operators	Modified Schrödinger	Guidance equation	Advanced/retarded waves
Hilb status	Assumed	Assumed	Modified	Assumed	Assumed
Consistency with QP	Complementary	Partially subsumed	Incompatible (modifies QM)	Incompatible (hidden vars.)	Partially compatible

11 Synthesis: QP as a Meta-Framework

11.1 The Landscape of Interpretations, Unified

The systematic comparison of the previous sections reveals a striking pattern: each existing interpretive framework captures a *partial truth* about quantum mechanics, and Quantum Perspectivism provides the meta-framework that explains why each partial truth is partial and how they fit together.

1. **RQM captures the relational truth**: quantum states are relative to observers. QP grounds this in the Yoneda Lemma.
2. **QBism captures the perspectival truth**: quantum mechanics is fundamentally about perspectives. QP objectifies this by replacing subjective belief with structural relation.
3. **Many-Worlds captures the unitary truth**: quantum evolution never collapses. QP preserves unitarity without ontological proliferation.
4. **Topos approaches capture the logical truth**: quantum logic is the internal logic of a topos. QP provides the physical motivation for the choice of topos.

5. **CQM captures the compositional truth**: quantum mechanics has a rich algebraic structure. QP derives this structure from the Yoneda Constraint.

6. **Consistent histories capture the temporal truth**: quantum mechanics can be formulated in terms of histories without collapse. QP embeds this in the temporal context category.

7. **Objective collapse theories capture the phenomenological truth**: macroscopic systems appear to collapse. QP explains this as decoherence from coarse-graining of contexts.

8. **Bohmian mechanics captures the deterministic truth**: there is a deterministic dynamics underlying the statistical predictions. QP replaces hidden trajectories with the deterministic presheaf, which is fully determined by the Yoneda Constraint.

11.2 Why QP Is Not "Just Another Interpretation"

One might object that QP is simply another entry in the already crowded field of quantum interpretations. We argue that this objection misunderstands the nature of QP. The standard interpretations (Copenhagen, Bohm, Many-Worlds, etc.) all take the mathematical formalism of quantum mechanics as given and then provide different philosophical glosses on that formalism. QP is fundamentally different: it *derives* the mathematical formalism from a single structural principle (the Yoneda Constraint) and, as a byproduct, resolves the interpretive puzzles that the standard formalism generates.

Proposition 11.1 (QP as Meta-Framework)**.** *QP stands in a different logical relationship to the standard interpretations than they stand to each other. Specifically:*

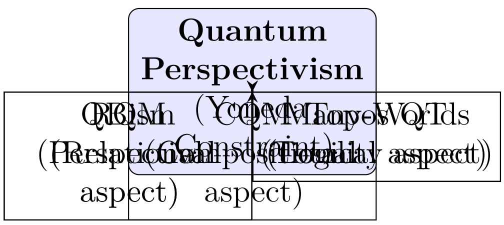
(a) Standard interpretations are interpretations of a given formalism*: they take* **Hilb***, the Born rule, etc., as given and attach physical/philosophical meaning.*

(b) QP is a derivation of the formalism*: it derives* **Hilb***, the Born rule, etc., from the Yoneda Constraint, and the physical/philosophical meaning emerges automatically as perspectival structure.*

(c) Each standard interpretation corresponds to a particular aspect *of the QP framework: RQM to the relational character of presheaves, QBism to the perspectival character of sections, Many-Worlds to the totality of contexts, etc.*

11.3 The Categorical Hierarchy of Interpretations

We can organize the interpretations into a categorical hierarchy based on their relationship to QP:



11.4 Open Questions for the Synthesis

The synthesis of QP with existing frameworks raises several open questions:

1. **Can CQM's graphical calculus be extended to the presheaf level?** Can we develop a string-diagrammatic calculus not just within **Hilb** but for presheaves on $\mathscr{C}$ with **Hilb**-valued fibers? This would unify the computational power of CQM with the foundational depth of QP.

2. **What is the correct context category for quantum gravity?** The IBD program uses $\mathcal{V}(\mathcal{H})$, the poset of abelian subalgebras. QP suggests a more general $\mathscr{C}$. For quantum gravity, the correct $\mathscr{C}$ may encode both quantum and gravitational degrees of freedom, providing a natural arena for unification.

3. **Can the consistent histories formulation be fully embedded in QP?** While we have sketched the embedding using temporal contexts, a complete treatment requires specifying the relationship between the decoherence functional and the presheaf inner product.

4. **Are there experimental signatures that distinguish QP from objective collapse theories?** Since GRW modifies quantum mechanics while QP preserves it, they make different predictions for mesoscopic systems. Developing these predictions in detail is important future work.

12 Conclusion

We have provided a comprehensive comparison of Quantum Perspectivism with the major existing frameworks in quantum foundations, spanning interpretive programs (RQM, QBism, Many-Worlds, consistent histories), mathematical programs (topos quantum theory, categorical quantum mechanics), and alternative theories (GRW, Bohm, transactional).

The central finding is that QP functions as a **meta-framework** that subsumes, clarifies, and extends the insights of each existing approach:

(i) **RQM** is subsumed: QP provides the mathematical backbone (presheaf functoriality) that RQM's relational ontology requires.

(ii) **QBism** is objectified: QP replaces subjective credences with objective presheaf sections while preserving the perspectival character of quantum states.

(iii) **Many-Worlds** is economized: QP preserves unitarity without ontological proliferation by treating branches as context-restrictions rather than real worlds.

(iv) **Topos QT** is motivated: QP provides the physical justification (the Yoneda Constraint) for the mathematical choice of a presheaf topos.

(v) **CQM** is grounded: QP derives the Hilbert space structure that CQM takes as its starting point.

(vi) **Consistent histories** are embedded: the consistent histories framework maps naturally into the temporal context category.

(vii) **Objective collapse** is reinterpreted: the appearance of collapse is a decoherence effect from coarse-graining of contexts, not a physical mechanism.

(viii) **Hidden variables** are excluded: the Yoneda Constraint's full faithfulness leaves no room for supplementary variables.

The Yoneda Lemma—the most fundamental theorem about mathematical identity—provides the single organizing principle from which all of these results flow. Quantum mechanics is not a collection of mysterious axioms but the unique physics of a world in which *to be is to be related.*

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